

# $[SU(3) \times SU(2) \times U(1)]^2$ and Strong Unification

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## Abstract

A supersymmetric model with gauge symmetry  $G_1 \times G_2$ , where  $G_i = SU(3)_i \times SU(2)_i \times U(1)_i$ , is constructed within the framework of gauge mediated supersymmetry breaking. At the energy scale  $\sim (10 - 100)$  TeV where the gauge symmetry breaks down to the Standard Model (SM),  $G_1$  is strong and  $G_2$  is weak. The observed gauge coupling constant unification of the SM is attributed to that of  $G_2$ . The messenger fields and Higgs fields just satisfy the condition that makes  $G_2$  a realization of strong unification. The SM gauginos are predicted to be generally heavier than the sleptons and squarks.

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Extensions of the SM aim at understanding new experimental results or unsolved theoretical problems. The most popular approach is the grand unification theories (GUTs) [1], such as the  $SU(5)$  GUT. There are indirect experimental evidences for GUTs from LEP and neutrino physics. To make GUTs viable, supersymmetry (SUSY) [2,3] is a must. One of the novel idea towards GUT is the so-called strong unification [4,5]. In the strong GUT, the SM gauge coupling constants just reach their common Landau pole at the unification energy scale.

Strong GUT is interesting not only due to its novelty, but also because of its usefulness. There is a discrepancy between the measured value of the QCD strong coupling constant at  $M_Z$ , which is  $\alpha_s^{\text{exp}}(M_Z) \simeq 0.1172 \pm 0.002$  [6], and that predicted by the minimal SUSY SM (MSSM)  $\alpha_s^{\text{MSSM}}(M_Z) \simeq 0.126$ . The discrepancy is reduced if extra matters are added into the MSSM. To keep the unification, the additional states should be in complete representation of GUT gauge groups. To the two-loop level, it has been shown [5], for example, that  $\alpha_s(M_Z) \simeq 0.1163$  if there are additional six multiplets in  $\mathbf{5} + \bar{\mathbf{5}}$  under  $SU(5)$  with masses of  $\sim 214$  TeV. However, the model would be artificial if these additional matters are naively added.

We will illustrate that a SUSY model with the gauge symmetry  $SU(3)_1 \times SU(2)_1 \times U(1)_1 \times SU(3)_2 \times SU(2)_2 \times U(1)_2$  can be a nontrivial realization of the strong GUT. There are multiple motivations to consider such an extension of the SM [7–9]. In Ref. [9], such kind of models were proposed as GUT generalization of the SUSY top-color [10]. They provide a solution to the SUSY flavor changing neutral current problem. However, the gauge coupling constant behavior was rather bad at high energies because of the introduction of too many extra matter fields which made the gauge interactions to be too much strong. This situation brings us to further think of their connection with the idea of the strong GUT. In this paper, after naturally modifying the Higgs and messenger contents of the model, we note that the extra matters additional to the MSSM can make the SM-like gauge interaction  $SU(3)_2 \times SU(2)_2 \times U(1)_2$  a strong GUT.

We consider a SUSY theory with the gauge group  $G_1 \times G_2$  in the framework of gauge

mediated SUSY breaking (GMSB) [11], where  $G_i = SU(3)_i \times SU(2)_i \times U(1)_i$  ( $i = 1, 2$ ). The three coupling constants of  $G_1$  are large, and those of  $G_2$  are small at the TeV scale. The three generations of matter carry nontrivial quantum numbers of  $G_2$  only. These numbers are assigned in the same way as they are under the SM gauge group. One gauge singlet chiral superfield  $X$  is introduced for SUSY breaking,

$$\langle X_s \rangle \neq 0, \quad \langle F_X \rangle \neq 0, \quad (1)$$

with  $X_s$  and  $F_X$  being the scalar and auxiliary components of  $X$ . The vacuum expectation values are taken to be real.

For the SUSY breaking messengers and gauge symmetry breaking Higgs', it is easy to consider them through imaging global  $SU(5)_i$  symmetry into which  $G_i$  is embedded. The messengers with their quantum numbers under  $SU(5)_1 \times SU(5)_2$  are

$$\begin{aligned} T_1(5, 1), \quad & \bar{T}_1(\bar{5}, 1), \\ T_2(1, 5), \quad & \bar{T}_2(1, \bar{5}) \end{aligned} \quad (2)$$

The relevant superpotential is

$$\mathcal{W}_1 = c_1 X T_1 \bar{T}_1 + c_2 X T_2 \bar{T}_2, \quad (3)$$

where  $c_1$  and  $c_2$  are coupling constants of order one. The fields  $T_i$  and  $\bar{T}_i$  are massive at tree level. Their fermionic components compose a Dirac fermion with mass  $c_i \langle X_s \rangle$ , while the scalar components have a squared-mass matrix

$$(T_{is}^* \quad \bar{T}_{is}) \begin{pmatrix} c_i^2 \langle X_s \rangle^2 & c_i \langle F_X \rangle \\ c_i \langle F_X \rangle & c_i^2 \langle X_s \rangle^2 \end{pmatrix} \begin{pmatrix} T_{is} \\ \bar{T}_{is}^* \end{pmatrix}. \quad (4)$$

The mass eigenstates and squared-mass eigenvalues are

$$\begin{aligned} \frac{1}{\sqrt{2}}(T_{is} + \bar{T}_{is}^*) & \quad \text{with} \quad m_{i1}^2 = c_i^2 \langle X_s \rangle^2 + c_i \langle F_X \rangle, \\ \frac{1}{\sqrt{2}}(T_{is} - \bar{T}_{is}^*) & \quad \text{with} \quad m_{i2}^2 = c_i^2 \langle X_s \rangle^2 - c_i \langle F_X \rangle. \end{aligned} \quad (5)$$

It is assumed that  $c_i \langle F_X \rangle < c_i^2 \langle X_s \rangle^2$ . Because  $\langle F_X \rangle \neq 0$ , SUSY breaking occurs in the fields  $T_i$ 's and  $\bar{T}_i$ 's at tree-level.  $G_1$  and  $G_2$  sectors get to be soft SUSY breaking via the

messengers at loop level. Because  $G_2$  is weak at TeV scale, its SUSY breaking effects can be calculated perturbatively, for example,  $G_2$  gaugino soft masses are

$$M_{\lambda'_r} \simeq \frac{\alpha'_r}{4\pi} \frac{\langle F_X \rangle}{\langle X_s \rangle}, \quad (6)$$

where  $\alpha'_r = g'_r/4\pi$  with  $g'_r$  being the gauge coupling constants of  $G_2$ . And  $r = 1, 2, 3$  corresponding to the groups  $U(1)$ ,  $SU(2)$ , and  $SU(3)$ , respectively. However,  $G_1$  is strong, we can only estimate its gaugino masses

$$M_{\lambda_r} \simeq \frac{\langle F_X \rangle}{\langle X_s \rangle}. \quad (7)$$

Numerically the messenger masses are about  $(10 - 100)$  TeV.

A pair of Higgs  $\Phi_1(5, \bar{5})$  and  $\Phi_2(\bar{5}, 5)$  breaks the  $G_1 \times G_2$  gauge symmetry down to that of the SM. One gauge singlet superfield  $Y$  is introduced for the gauge symmetry breaking. The superpotential of them is written as follows,

$$\mathcal{W}_2 = c' Y [\text{Tr}(\Phi_1 \Phi_2) - \mu'^2], \quad (8)$$

where the trace is taken with regard to both  $SU(3)_1 \times SU(3)_2$  and  $SU(2)_1 \times SU(2)_2$ .  $\mu'$  is the energy scale relevant to the gauge symmetry breaking, and  $c'$  is the coupling constant. The Higgs fields get soft masses like that given in Eq. (7). However, the above superpotential is not enough to guarantee all the  $\Phi_i$  fermion components to be massive. Their masses are nonvanishing when a superfield  $A$  which is in adjoint representation of  $SU(5)_1$  is introduced with the following superpotential [8],

$$\mathcal{W}'_2 = c'_2 \text{Tr}(\Phi_2 A \Phi_1) \quad (9)$$

with  $c'_2$  being the coupling constant. The details of the gauge symmetry breaking go the same way as that in Ref. [9] (its Eqs. (10-17)). The VEVs of the  $\Phi_i$ 's are given as

$$\langle \Phi_{1s} \rangle = \langle \Phi_{2s} \rangle = v I_3 \otimes I_2, \quad (10)$$

where  $I_3$  and  $I_2$  are the unit matrices in the space of  $SU(3)_1 \times SU(3)_2$  and  $SU(2)_1 \times SU(2)_2$ , respectively. The coupling constants of the SM  $SU(3)_c \times SU(2)_L \times U(1)_Y$  are

$$\frac{1}{g_s^2} = \frac{1}{g_3^2} + \frac{1}{g_3'^2}, \quad \frac{1}{g^2} = \frac{1}{g_2^2} + \frac{1}{g_2'^2}, \quad \frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_1'^2}. \quad (11)$$

Numerically, the gauge symmetry breaking scale  $v$  is about  $(10 - 100)$  TeV.

Electroweak symmetry breaking is achieved via a pair of Higgs superfields  $H_u$  and  $H_d$  which are nontrivial only under  $G_2$  [9].

Around  $10 - 100$  TeV, there are many matter fields which will run the gauge coupling constants to be large at high energies. The matter fields introduced additional to MSSM are complete  $SU(5)$  multiplets. Therefore, the unification scale  $3 \times 10^{16}$  GeV is still the same as that of the MSSM, but the values of the coupling constants are significantly different. This model is a candidate of strong GUT.

Below the scale  $v$ , the  $G_1 \times G_2$  breaks spontaneously down to the MSSM. From Eq. (11), it is easy to see that the gauge coupling constants of MSSM are almost fully determined by that of  $G_2$ , because  $g_i \gg g'_i$ . Therefore the observed unification of MSSM is attributed to the unification of  $G_2$ .

Above  $(10 - 100)$  TeV scale, the theory is  $G_1 \times G_2$ . As far as the  $G_2$  sector is concerned, the new matter fields in addition to the MSSM are the messengers  $T_1$  and  $\bar{T}_1$ , and Higgs fields  $\Phi_1$  and  $\Phi_2$ . The messenger fields compose one  $\mathbf{5} + \bar{\mathbf{5}}$  multiplet with a mass  $c\langle X_s \rangle$  and the Higgs' contribute five  $\mathbf{5} + \bar{\mathbf{5}}$  multiplets with masses  $c'v$  as well as  $c'_2v$ . We have the freedom to adjust all the masses of these six  $\mathbf{5} + \bar{\mathbf{5}}$  multiplets to be about 214 TeV. As has been shown in Ref. [5], the gauge couplings reach their common Landau pole at the GUT scale  $\sim 3 \times 10^{16}$  GeV. Namely in this case,  $G_2$  is a realization of the strong GUT.

Some remarks are necessary. (1) The perturbative calculation in Ref. [5] was not reliable around the GUT scale because of the large coupling constants. But around 100 TeV where the perturbative domain lies, its reliability was under control. It is in the latter low energy region where we have made use of Ref. [5]. (2) On the other hand,  $G_1$  sector is also expected to be a GUT.  $(10 - 100)$  TeV is already its non-perturbative region, we have no reliable method yet to make detailed analysis. (3) The unification simply means that the gauge coupling constants are equal at certain scales. We have not introduced any unified gauge

group. Such a model does not have proton decays, and does not suffer from the doublet-triplet splitting problem. (4) It should be noted that only is  $G_2$  SM-like, can the breaking  $G_1 \times G_2 \rightarrow \text{SM}$  at  $(10 - 100)$  TeV occur. Any breaking of  $SU(5) \times SU(5) \rightarrow SU(5)$  [12] would have occurred above  $3 \times 10^{16}$  GeV. (5) Some of the matter contents of  $G_2$ , such as the third generation can be moved into  $G_1$ . Due to GMSB, the superpartners in this sector are very heavy  $\sim 10 - 100$  TeV. They decouple at  $(1 - 10)$  TeV energy scale. At this low energy scale the fermions, on the other hand, can form condensates due to the strong gauge interactions. Dynamical fermion masses might be generated [10]. In order to keep the strong GUT, it is possible to either introduce one more  $\mathbf{5} + \bar{\mathbf{5}}$  multiplet of  $G_2$ , which may play a role of SUSY breaking messengers [9], or lower the SUSY breaking and messenger scales to be around 10 TeV. These possibilities should be studied further and are beyond the scope of this work. (6) If the  $SU(3)_1$  interaction is switched off, the model is a kind of top-flavor models [13].

This model has interesting phenomenology. Besides the new gauge bosons, gauginos and Higgs particles with masses around  $(10 - 100)$  TeV, the SM gaugino masses are predicted to be as heavy as  $\sim 100$  GeV – 1 TeV. Let us analyze the gaugino spectrum in more detail. The full gaugino masses have two origins: SUSY breaking (soft masses) and spontaneous gauge symmetry breaking. It has been obtained in Ref. [9] that the relevant mass matrix in the basis of  $\lambda_r$ ,  $\lambda'_r$  and the higgsino  $(\psi_1 - \psi_2)/\sqrt{2}$  is

$$M_r = \begin{pmatrix} M_{\lambda_r} & 0 & \sqrt{2}g_r v \\ 0 & M_{\lambda'_r} & \sqrt{2}g'_r v \\ \sqrt{2}g_r v & \sqrt{2}g'_r v & 0 \end{pmatrix}, \quad (12)$$

where  $\psi_1$  and  $\psi_2$  stand for the fermion components of  $\Phi_1$  and  $\Phi_2$ , respectively. Numerically at the scale  $v \sim (10 - 100)$  TeV,  $g'_r \sim 0.1$ ,  $g_r \sim 1$ . The mass matrix determines two heavy states with masses  $\sim M_{\lambda_r} \sim g_r v \sim (10 - 100)$  TeV, and one lighter state  $\sim (g'_r v)^2/M_{\lambda_r} \sim 100$  GeV – 1 TeV. This lighter state is a mixture of the  $G_2$  gaugino with the higgsino. It is regarded as the MSSM gaugino in this model. On the other hand, the soft masses of the three generation matters are about 100 GeV. Therefore in this model the SM gauginos

are generally heavier than the sleptons and squarks. Such a mass pattern can be tested in future colliders.

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